11.8

POWER SERIES $= \sum_{n=0}^{\infty} c_n (x-a)^n$ $\quad c_n \in \mathbb{R}$

DEFINE $0^0 = 1$
FOR POWERSERIES

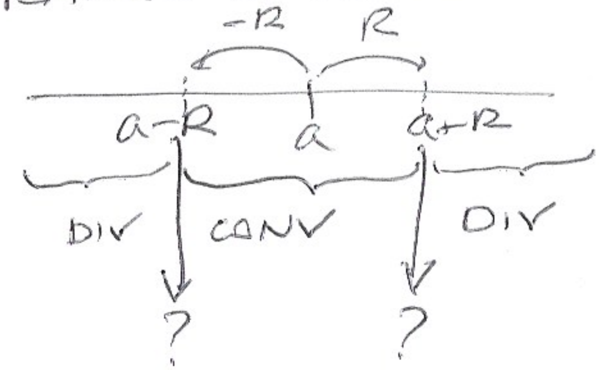
TO FIND INTERVAL OF CONVERGENCE (IE. ALL X-VALUES FOR WHICH THE SERIES CONVERGES)

① USE RATIO TEST: SOLVE $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

(GET $|x-a| < R$) ← RADIUS OF CONVERGENCE

$-R < x-a < R$

$a-R < x < a+R$



NOT RATIO TEST

② USE OTHER TESTS TO DETERMINE IF SERIES CONVERGES AT ENDS OF RADII @ $x = a+R$ AND @ $x = a-R$

DETERMINE WHERE $\sum_{n=0}^{\infty} \underbrace{2^{n^2}(x-5)^n}_{a_n}$ CONV.

$\downarrow$

FOR WHICH X-VALUES

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{2^{(n+1)^2}(x-5)^{n+1}}{2^{n^2}(x-5)^n}\right| = \lim_{n\to\infty}\left|2^{2n+1}(x-5)\right|$$

ONLY VALID IF $\lim_{n\to\infty} 2^{2n}$ EXISTS

$$= |x-5| \lim_{n\to\infty} 2^{2n+1} \longrightarrow \infty$$

IF $X \neq 5$

IS NEVER < 1 IF $X \neq 5$

IE. SERIES DIV IF $X \neq 5$

IF $X = 5$

$$\lim_{n\to\infty}\left|2^{2n+1}\cdot 0\right| = \lim_{n\to\infty} 0 = 0 < 1$$

IE, $\boxed{\text{SERIES CONV IF } X=5}$

$R = 0$

DETERMINE WHERE $(\sin x =)\ x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$n=0 \quad 1 \quad 2 \quad 3$

$\underline{0} + x + \underline{0}x^2 - \frac{x^3}{3!} + \underline{0}x^4 + \frac{x^5}{5!} + \underline{0}x^6 - \frac{x^7}{7!} \cdots$

$$\sum_{n=0}^{\infty} \underbrace{\frac{(-1)^n x^{2n+1}}{(2n+1)!}}_{a_n}$$

1, 3, 5, 7
EXPONENTS = ARITHMETIC SEQUENCE

$$\cancel{\begin{aligned} a_n &= a_1 + (n-1)d \leftarrow \text{START } n=1 \\ &= 1 + (n-1)2 \\ &= 2n - 2 + 1 \\ &= 2n - 1 \end{aligned}}$$

$a_0 = 2(0) - 1 = -1 \neq 1$

$$\begin{aligned} a_n &= a_0 + nd \leftarrow \text{START } n=0 \\ &= 1 + n(2) \\ &= 2n+1 \end{aligned}$$

SANITY CHECK: $n=3$

$2n+1 = 7$ ✓

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{2(n+1)+1}}{(2(n+1)+1)!} \frac{(2n+1)!}{(-1)^n x^{2n+1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{(-1)^n} \cdot \frac{x^{2n+3}}{x^{2n+1}} \cdot \frac{(2n+1)!}{(2n+3)!} \right|$$

$$= \lim_{n\to\infty} \left| \cancel{(-1)}\, x^2 \frac{1}{(2n+3)(2n+2)} \right|$$

$$\frac{(2n+1)!}{(2n+3)!} = \frac{1\ \cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}}$$

$-1 \quad -1$

$$= x^2 \lim_{n\to\infty} \frac{1}{(2n+3)(2n+2)}$$

$$= 0 < 1$$

CONV

0

FOR ALL $x \in \mathbb{R}$, $0 < 1$

SO $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ CONV ON $(-\infty, \infty) \rightarrow R = \infty$

FIND RADIUS + INTERVAL OF CONVERGENCE OF $\displaystyle\sum_{n=1}^{\infty} \frac{(x+3)^{2n}}{9^{n+1}}$

CENTERED ABOUT $x=-3$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x+3)^{2n+2}}{9^{n+2}}\cdot\frac{9^{n+1}}{(x+3)^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x+3)^2}{9}\right| = \left|\frac{(x+3)^2}{9}\right| < 1$$

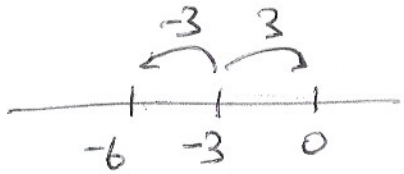
$$\frac{(x+3)^2}{9} < 1$$

$$(x+3)^2 < 9$$

$$-3 < x+3 < 3$$

$$-6 < x < 0$$

IF $a>0$,
$x^2 < a \longleftrightarrow$
$-\sqrt{a} < x < \sqrt{a}$



INTERVAL: $(-6,0)$ + ~~MAYBE ENDPOINTS~~?

RADIUS: $3 = \dfrac{0 - -6}{2}$

$$x=0:\ \sum_{n=1}^{\infty}\frac{(3)^{2n}}{9^{n+1}} = \sum_{n=1}^{\infty}\frac{9^n}{9^{n+1}} = \sum_{n=1}^{\infty}\frac{1}{9} \qquad \lim_{n\to\infty}\frac{1}{9} = \frac{1}{9} \neq 0 \ \ \underline{\text{DIV}}\ (\text{DIV TEST})$$

$$x=-6:\ \sum_{n=1}^{\infty}\frac{(-3)^{2n}}{9^{n+1}} = \sum_{n=1}^{\infty}\frac{(-1)^{2n}(3)^{2n}}{9^{n+1}} = \sum_{n=1}^{\infty}\frac{1}{9}$$